$$n = \frac{b}{\sqrt{-4c}}, \quad 4m^2 - 1 = 2a + a^2$$

Taking account of the first boundary condition in (4), the general solution of (12) is

$$T_2 = \theta^{-a/2}\,[C_1 M_{n,m}(\xi) + C_2 W_{n,m}(\xi)]\sin R\varphi$$

$$M_{n,m}(\xi) = e^{-\xi/2}\,\xi^{m+1/2}\,\Phi\,(m - n + 1/2,\ 2m + 1,\ \xi)$$

$$W_{n,m}(\xi) = e^{-\xi/2}\,\xi^{m+1/2}\,\Psi\,(m - n + 1/2,\ 2m + 1,\ \xi)$$

(Φ, Ψ are degenerate hypergeometric functions). The diagram of T_2 in the section $\theta = -0.4$, obtained by this method, is presented in Fig. 3 (curve 3).

The bending state of stress in the sector of a thin toroidal shell segment corresponds to the character of the change in the membrane forces and is determined on the foundation of the membrane solution obtained. Knowing the general character of the change in the state of stress and the magnitude of the moments in the section $\varphi = \varphi_0$ (from the solution of the known problem for a shell closed along the circumferential coordinate), the magnitude of the bending moments can be determined in the section $0 \leqslant \varphi \leqslant \varphi_0$.

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BOUNDARY VALUE PROBLEMS FOR AN ELASTIC ANISOTROPIC HALF-PLANE WEAKENED BY A CIRCULAR HOLE

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A method is given for solving some boundary value problems for a half-plane with a circular hole. It is assumed that the material of the half-plane possesses rectilinear anisotropy of a general kind and that planes of symmetry perpendicular to the O-axis exist. The half-plane is weakened by a circular hole L_1 of unit radius subjected to an internal pressure p. The affix of the center of L_1 (Fig. 1) is denoted by $\bar{a}$

1. The solution of the problem consists of seeking two analytic functions $\Phi_j(z_j)$ ($j=1, 2$) in the appropriate domains, in whose terms the stress and displacement components are expressed [1]. Here $z_j = x + \mu_j y$ are generalized complex variables, and μ_1, μ_2 are roots of a certain characteristic equation.

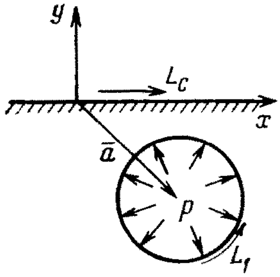

Fig. 1

The stress components on an area with normal n are determined by means of the formulas

$$\sigma_n = \delta_1^2\Phi_1'(z_1) + \bar{\delta}_1^2\overline{\Phi_1'(z_1)} + \delta_2^2\Phi_2'(z_2) + \bar{\delta}_2^2\overline{\Phi_2'(z_2)} \tag{1.1}$$

$$\tau_n = \delta_1\gamma_1\Phi_1'(z_1) + \bar{\delta}_1\bar{\gamma}_1\overline{\Phi_1'(z_1)} + \delta_2\gamma_2\Phi_2'(z_2) + \bar{\delta}_2\bar{\gamma}_2\overline{\Phi_2'(z_2)}$$

$$\delta_j = \cos(n, y) - \mu_j\cos(n, x), \quad \gamma_j = \cos(n, y) + \mu_j\cos(n, x)$$

A half-plane S_j^-, bounded by the same line L_0 and weakened by an elliptical hole γ_j with center at the point $\bar{a}_j$ corresponds in the complex z_j-plane to the half-plane S^- weakened by the circular hole L_1.

The function mapping the exterior of the unit circle on the exterior of the ellipse is

$$z_j = \bar{a}_j + {}^1/_2\; c_j e^{i\theta_j}[\rho_j\zeta_j + (\rho_j\zeta_j)^{-1}]$$

$$\rho_j = \sqrt{\frac{b_j + d_j}{b_j - d_j}}, \quad c_j = \sqrt{b_j^2 - d_j^2}$$

Here b_j, d_j are the major and minor semi-axes of the ellipse, θ_j is the slope of the major semi-axis to Ox. The inverse function is represented as follows:

$$\zeta_j = \chi_j(z_j) = \frac{z_j - \bar{a}_j + \sqrt{(z_j - \bar{a}_j)^2 - c_j^2 e^{2i\theta_j}}}{\rho_j c_j e^{i\theta_j}}$$

The relationships

$$\delta_j(t_j) = \frac{1}{2}(i + \mu_j)\,\chi_j^{-1}(t_j)\left(\frac{i - \mu_j}{i + \mu_j} - \chi_j^2(t_j)\right)$$

$$\gamma_j(t_j) = \frac{1}{2}(i - \mu_j)\,\chi_j^{-1}(t_j)\left(\frac{i + \mu_j}{i - \mu_j} - \chi_j^2(t_j)\right)$$

are valid on the contour of the unit circle L_1. Here the point $\chi_j(t_j)$ belongs to L_1 and t_j is a point of γ_j.

2. F i r s t b o u n d a r y v a l u e p r o b l e m. The normal $N(t_0)$ and tangential $T(t_0)$ forces summed over L_0 are given on the rectilinear boundary of the half-plane L_0. We then have

$$\Phi_1'(t_0) + \Phi_2'(t_0) + \overline{\Phi_1'(t_0)} + \overline{\Phi_2'(t_0)} = -N(t_0) \quad \text{on } L_0 \tag{2.1}$$

$$\mu_1\Phi_1'(t_0) + \mu_2\Phi_2'(t_0) + \bar{\mu}_1\overline{\Phi_1'(t_0)} + \bar{\mu}_2\overline{\Phi_2'(t_0)} = T(t_0) \quad \text{on } L_0 \tag{2.2}$$

$$\xi_1(t_1)\Phi_1'(t_1) + \xi_2(t_2)\Phi_2'(t_2) + \eta_1(t_1)\overline{\Phi_1'(t_1)} + \eta_2(t_2)\overline{\Phi_2'(t_2)} = -p \quad \text{on } L_1 \tag{2.3}$$

$$\xi_j(t_j) = \delta_j(t_j)(\delta_j(t_j) + i\gamma_j(t_j)), \quad \eta_j(t_j) = \overline{\delta_j(t_j)}\,\overline{(\delta_j(t_j) + i\gamma_j(t_j))}$$

Let us introduce unknown auxiliary functions on L_1

$$2\xi_1(t_1)\,\omega_1(t) = \xi_1(t_1)\Phi_1'(t_1) - \xi_2(t_2)\,\Phi_2'(t_2) - \eta_1(t_1)\,\overline{\Phi_1'(t_1)} - \eta_2(t_2)\,\overline{\Phi_2'(t_2)} \tag{2.4}$$

$$2\xi_2(t_2)\,\omega_2(t) = -\xi_1(t_1)\,\Phi_1'(t_1) + \xi_2(t_2)\,\Phi_2'(t_2) - \eta_1(t_1)\,\overline{\Phi_1'(t_1)} - \eta_2(t_2)\,\overline{\Phi_2'(t_2)} \tag{2.5}$$

Adding (2.3) to (2.4) and (2.3) to (2.5), respectively, we obtain

$$\Phi_j'(t_j) = \omega_j(t) - \frac{p}{2\xi_j(t_j)} \qquad (j = 1, 2) \tag{2.6}$$

On the basis of the properties of Cauchy-type integrals, we find from (2.6) that the regular function in the domain outside the ellipse γ_j

$$\Phi_{j*}'(z_j) = \Phi_j'(z_j) + I_{1j}(z_j) - I_{2j}(z_j) \tag{2.7}$$

$$I_{1j}(z_j) = \frac{1}{2\pi i}\int_{\gamma_j} \frac{\omega_j(t)\,dt_j}{t_j - z_j}; \qquad I_{2j}(z_j) = \frac{p}{2}\,\frac{1}{2\pi i}\int_{\gamma_j} \frac{\xi_j^{-1}(t_j)\,dt_j}{t_j - z_j}$$

is analytically continuable within the ellipse γ_j.

Let us substitute (2.7) into (2.1) and (2.2), then we obtain for the regular functions $\Phi_{j*}'(z_j)$ in the whole lower half-plane

$$\Phi_{1*}'(t_0) + \Phi_{2*}'(t_0) + \overline{\Phi_{1*}'(t_0)} + \overline{\Phi_{2*}'(t_0)} = -N(t_0) + R_*(t_0) \tag{2.8}$$
$$\mu_1\Phi_{1*}'(t_0) + \mu_2\Phi_{2*}'(t_0) + \bar\mu_1\overline{\Phi_{1*}'(t_0)} + \bar\mu_2\overline{\Phi_{2*}'(t_0)} = T(t_0) + Q_*(t_0)$$

$$R_*(t_0) = I_{11}(t_0) - I_{21}(t_0) + I_{12}(t_0) - I_{22}(t_0) + \overline{I_{11}(t_0)} - \overline{I_{21}(t_0)} + \overline{I_{12}(t_0)} - \overline{I_{22}(t_0)} \tag{2.9}$$

$$Q_*(t_0) = \mu_1[I_{11}(t_0) - I_{21}(t_0)] + \mu_2[I_{12}(t_0) - I_{22}(t_0)] + \bar\mu_1[\overline{I_{11}(t_0)} - \overline{I_{21}(t_0)}] + \bar\mu_2[\overline{I_{12}(t_0)} - \overline{I_{22}(t_0)}]$$

We multiply (2.8) by a Cauchy kernel and integrate over L_0, then

$$\Phi_{1*}'(z_1) = \frac{1}{\mu_2 - \mu_1}[\varphi_1(z_1) - \varphi_2(z_1)], \quad \Phi_{2*}'(z_1) = \frac{-1}{\mu_2 - \mu_1}[\psi_1(z_2) - \psi_2(z_2)]$$

$$\varphi_1(z_1) = \frac{1}{2\pi i}\int_{L_0} \frac{\mu_2 N(t_0) + T(t_0)}{t_0 - z_1}\,dt_0, \quad \varphi_2(z_1) = \frac{1}{2\pi i}\int_{L_0} \frac{\mu_2 R_*(t_0) - Q_*(t_0)}{t_0 - z_1}\,dt_0$$

$$\psi_1(z_2) = \frac{1}{2\pi i}\int_{L_0} \frac{\mu_1 N(t_0) + T(t_0)}{t_0 - z_2}\,dt_0, \quad \psi_2(z_2) = \frac{1}{2\pi i}\int_{L_0} \frac{\mu_1 R_*(t) - Q_*(t_0)}{t_0 - z_2}\,dt_0$$

3. Mixed boundary value problem. A stamp with a flat base is pressed by a force P on a half-plane with a circular hole. The presence of a dry friction force ($T = \rho_* P$) is assumed between the stamp and the boundary L_0 It is also assumed that the stamp is displaced translationally. Then we have on L_0

$$\Phi_1'(t_0) + \Phi_2'(t_0) + \overline{\Phi_1'(t_0)} + \overline{\Phi_2'(t_0)} = 0 \tag{3.1}$$

$$\mu_1\Phi_1'(t_0) + \mu_2\Phi_2'(t_0) + \bar\mu_1\overline{\Phi_1'(t_0)} + \bar\mu_2\overline{\Phi_2'(t_0)} = 0, \quad |t_0| > l$$

$$q_1\Phi_1'(t_0) + q_2\Phi_2'(t_0) + \bar q_1\overline{\Phi_1'(t_0)} + \bar q_2\overline{\Phi_2'(t_0)} = 0$$
$$[\mu_1\Phi_1'(t_0) + \mu_2\Phi_2'(t_0) + \bar\mu_1\overline{\Phi_1'(t_0)} + \bar\mu_2\overline{\Phi_2'(t_0)}] = \rho_*[\Phi_1'(t_0) + \Phi_2'(t_0) + \overline{\Phi_1'(t_0)} + \overline{\Phi_2'(t_0)}], \quad |t_0| < l$$

The constants q_1, q_2 are expressed in terms of the constants of the material [1]. Proceeding from the functions $\Phi_{j*}'(z_j)$ constructed in Sect. 2, let us reduce this problem to the problem of a stamp impressed on a solid half-plane. Taking (2.7) into account and representing $\Phi_{j*}'(z_j)$ as

$$\Phi_{j*}'(z_j) = \varphi_{j*}'(z_j) + \psi_{j*}'(z_j)$$

where $\psi_{j*}'(z_j)$ is the solution of the first boundary value problem for a solid anisotropic half-plane when certain forces $R_*(t_0)$, $Q_*(t_0)$, are applied to the half-plane boundary, and we have for $\varphi_{j*}'(z_j)$ from (3.1)

$$\begin{gathered}
\varphi_{1*}'(t_0) + \varphi_{2*}'(t_0) + \overline{\varphi_{1*}'(t_0)} + \overline{\varphi_{2*}'(t_0)} = 0 \\
\mu_1\varphi_{1*}'(t_0) + \mu_2\varphi_{2*}'(t_0) + \bar{\mu}_1\overline{\varphi_{1*}'(t_0)} + \bar{\mu}_2\overline{\varphi_{2*}'(t_0)} = 0, \ |t_0| > l \\
q_1\varphi_{1*}'(t_0) + q_2\varphi_{2*}'(t_0) + \bar{q}_1\overline{\varphi_{1*}'(t_0)} + \bar{q}_2\overline{\varphi_{2*}'(t_0)} = M'(t_0) \\
[\mu_1\varphi_{1*}'(t_0) + \mu_2\varphi_{2*}'(t_0) + \bar{\mu}_1\overline{\varphi_{1*}'(t_0)} + \bar{\mu}_2\overline{\varphi_{2*}'(t_0)}] = \\
\rho_*[\varphi_{1*}'(t_0) + \varphi_{2*}'(t_0) + \overline{\varphi_{1*}'(t_0)} + \overline{\varphi_{2*}'(t_0)}], \ |t_0| < l \\
M'(t_0) = M_0'(t_0) - q_1\psi_{1*}'(t_0) - q_2\psi_{2*}'(t_0) - \bar{q}_1\overline{\psi_{1*}'(t_0)} - \bar{q}_2\overline{\psi_{2*}'(t_0)} \\
M_0(t_0) = q_1[I_{11}(t_0) - I_{21}(t_0)] + q_2[I_{12}(t_0) - I_{22}(t_0)] + \\
\bar{q}_1\overline{[I_{11}(t_0) - I_{21}(t_0)]} + \bar{q}_2\overline{[I_{12}(t_0) - I_{22}(t_0)]}
\end{gathered} \tag{3.2}$$

Following [2], we represent the analytic functions $\varphi_{j*}'(z_j)$ in the lower half-plane

$$\varphi_{1*}'(z_1) = -\frac{\mu_2 + \rho_*}{2\pi i(\mu_1 - \mu_2)} w_1(z_1), \qquad \varphi_{2*}'(z_2) = \frac{\mu_1 + \rho_*}{2\pi i(\mu_1 - \mu_2)} w_1(z_2)$$

To determine $w_1(z)$, a Riemann-Hilbert problem is constructed

$$w_1^+(t_0) + \frac{\varkappa_1 + i\varkappa_2}{\varkappa_1 - i\varkappa_2} w_1^-(t_0) = \frac{2M'(t_0)}{\varkappa_1 - i\varkappa_2}, \quad |t_0| < l$$

$$w_1^+(t_0) - w_1^-(t_0) = 0, \ |t_0| > l$$

$$\varkappa_1 = \frac{1}{\pi}(A_3 + \rho_* A_4), \quad \varkappa_2 = \frac{1}{\pi}(B_3 + \rho_* B_4)$$

Here A_3, A_4, B_3, B_4 are expressed in terms of the constants of the material [2].

The solution of the problem obtained is represented as

$$w_1(z) = \frac{X_0(z)}{\pi i(\varkappa_1 - i\varkappa_2)} \int_{-l}^{l} \frac{M'(t_0)\,dt_0}{X_0^+(t_0)(t_0 - z)} + C_1 X_0(z)$$

$$X_0(z) = (z + l)^{-\gamma^\circ}(z - l)^{\gamma^\circ - 1}, \qquad \gamma^\circ = \frac{1}{2\pi i}\ln\left(-\frac{\varkappa_1 + i\varkappa_2}{\varkappa_1 - i\varkappa_2}\right)$$

4. Let total adhesion hold under the stamp, and let the boundary outside the stamp be force-free; then

$$\begin{gathered}
\Phi_1'(t_0) + \Phi_2'(t_0) + \overline{\Phi_1'(t_0)} + \overline{\Phi_2'(t_0)} = 0 \\
\mu_1\Phi_1'(t_0) + \mu_2\Phi_2'(t_0) + \bar{\mu}_1\overline{\Phi_1'(t_0)} + \bar{\mu}_2\overline{\Phi_2'(t_0)} = 0, \ |t_0| > l \\
p_1\Phi_1'(t_0) + p_2\Phi_2'(t_0) + \bar{p}_1\overline{\Phi_1'(t_0)} + \bar{p}_2\overline{\Phi_2'(t_0)} = 0 \\
q_1\Phi_1'(t_0) + q_2\Phi_2'(t_0) + \bar{q}_1\overline{\Phi_1'(t_0)} + \bar{q}_2\overline{\Phi_2'(t_0)} = 0, \ |t_0| < l
\end{gathered} \tag{4.1}$$

Substituting (2.7) into (4.1), we obtain

$$\Phi_{1*}'(t_0) + \Phi_{2*}'(t_0) + \overline{\Phi_{1*}'(t_0)} + \overline{\Phi_{2*}'(t_0)} = R_*(t_0)$$

$$\mu_1 \Phi_{1*}'(t_0) + \mu_2\Phi_{2*}'(t_0) + \bar{\mu}_1\overline{\Phi_{1*}'(t_0)} + \bar{\mu}_2\overline{\Phi_2'(t_0)} = Q_*(t_0), \ |t_0| > l$$
$$p_1 \Phi_{1*}'(t_0) + p_2\Phi_{2*}'(t_0) + \bar{p}_1\overline{\Phi_{1*}'(t_0)} + \bar{p}_2\overline{\Phi_{2*}'(t_0)} = N_0(t_0)$$
$$q_1\Phi_{1*}'(t_0) + q_2\Phi_{2*}'(t_0) + \bar{q}_1\overline{\Phi_{1*}'(t_0)} + \bar{q}_2\overline{\Phi_{2*}'(t_0)} = M_0(t_0), \ |t_0| < l$$
$$N_0(t_0) = p_1[I_{11}(t_0) - I_{21}(t_0)] + p_2[I_{12}(t_0) - I_{22}(t_0)] + \bar{p}_1\overline{[I_{11}(t_0) - I_{21}(t_0)]} + \bar{p}_2\overline{[I_{12}(t_0) - I_{22}(t_0)]}$$

where $R_*(t_0)$, $Q_*(t_0)$ and $M_0(t_0)$ are determined from (2.9) and the last formula in (3.2).

These conditions correspond to a mixed boundary value problem for a solid half-plane. Following [2], we find

$$\Phi_{1*}'(z_1) = \frac{(1 - \bar{S}\mu_2)\, w_2(z_1) - (1 - S\mu_2)\, w_3(z_1)}{2\pi i\,(\mu_1 - \mu_2)(\bar{S} - S)}$$
$$\Phi_{2*}'(z_2) = \frac{(1 - \bar{S}\mu_1)\, w_2(z_2) - (1 - S\mu_1)\, w_3(z_2)}{2\pi i\,(\mu_1 - \mu_2)(\bar{S} - S)}$$

Riemann-Hilbert problems are constructed for the functions $w_2(z)$ and $w_3(z)$

$$w_j^+(t_0) + \frac{K_j + iQ_j}{K_j - iQ_j}\, w_j^-(t_0) = \frac{2\pi}{K_j - iQ_j}\,[N_0(t_0) + \lambda_j M_0(t_0)], \ |t_0| < l$$
$$w_j^+(t_0) - w_j^-(t_0) = 2\pi i\,[R_*(t_0) + S_j Q_*(t_0)], \quad |t_0| > l$$
$$K_2 = \bar{K}_3 = K, \quad Q_2 = \bar{Q}_3 = Q, \quad S_2 = \bar{S}_3 = S, \quad \lambda_2 = \bar{\lambda}_3 = \lambda$$

(the constants K, Q, λ, S are expressed in terms of constants of the material [2]). Solutions of these problems are

$$w_j(z) = \frac{X_{0j}(z)}{2\pi i}\int_{L_0} \frac{f_j(t_0)\, dt_0}{X_{0j}^+(t_0)\,(t_0 - z)} + C_j X_{0j}(z), \quad j = 2, 3$$

$$f_j(t_0) = \begin{cases} 2\pi i\,[R_*(t_0) + S_j Q_*(t_0)], & |t_0| > l \\ \dfrac{2\pi}{K_j - iQ_j}\,[N_0(t_0) + S_j M_0(t_0)], & |t_0| < l \end{cases}$$

$$X_{0j}(z) = (z + l)^{-\gamma_j^\circ}(z - l)^{\gamma_j^\circ - 1}, \quad \gamma_j^\circ = \frac{1}{2\pi i}\ln\left(-\frac{K_j + iQ_j}{K_j - iQ_j}\right)$$

The constants C_j are determined from the limit conditions for $w_j(z_j)$ at infinity [2]; they equal the force P acting on the stamp. To find $\omega_j(t)$ we represent them as the Fourier series

$$\omega_j(t) = \sum_{\nu=-\infty}^{\infty} \alpha_{\nu j}\,(t - \bar{a})^{\nu}.$$

Using this representation, we express the functions $\Phi_j'(z_j)$ as an infinite series in whose coefficients enter $\alpha_{\nu j}$. Two infinite systems of complex linear algebraic equations are obtained to determine them.

Numerical example. A computation is performed for the first fundamental problem with the following data: $p = 0$; $N(t_0) = 0$ for $|t_0| > l$ and $N(t_0) = 1$ for $|t_0| < l$, $T(t_0) = 0$, $a_0 = -ih$, $\mu_1 = 1.48i$, $\mu_2 = 0.53i$, $l = 1.2$, $h = 1.2$. Eleven equations are taken from the four infinite systems of real linear algebraic equations. Diagrams of the annular stresses at points of the circle are obtained. Values of the annular stresses at points of the circle are

θ	0	40	80	120	180
σ_θ	+10.87	+8.06	+6.97	−4.55	−1.58
σ_θ	+ 5.52	+3.96	+2.84	−3.12	−4.28

Values of σ_θ for the isotropic case are given in the third line; θ is the angle measured from the normal to the half-plane boundary.

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AXISYMMETRIC IMPRESSION OF TWO STAMPS INTO AN ELASTIC SPHERE

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The problem of the impression of two identical axisymmetric stamps in an elastic sphere is considered. It is assumed that the surface of the sphere outside the stamps is stress-free, while there are no shear stresses under the stamps. A solution is obtained for arbitrary stamps for both given and unknown in advance boundaries of the contact domains by the method elucidated in [1]. A numerical calculation is presented for spherical stamps under internal contact with the sphere.

The contact problem for a sphere in such a formulation (when the boundaries of the contact domains are known) was first studied in [2]. The problem was reduced to determining certain coefficients from dual series-equations containing Legendre polynomials. The method permitting reduction of the solution of the obtained dual series-equations to the solution of an infinite system of linear algebraic equations is indicated. This method is reduced to an integral equation of the first kind in [3] and a possible scheme is indicated for the approximate solution of the equation obtained.

1. Let us consider the contact problem of impressing two axisymmetric stamps (Fig. 1), whose surface is given in a spherical r, θ, φ coordinate system by the equation

$$r = R\,[1 + \rho(\theta)],\ \rho(\pi - \theta) = \rho(\theta),\ \rho(0) = 0 \tag{1.1}$$

onto an elastic sphere $r \leqslant R$.

The boundary conditions (on the sphere $r = R$) are ($2aR$ is the approach of the stamps):

$$u_r = R\,[-a\,|\cos\theta| + \rho(\theta)],\ 0 \leqslant \theta \leqslant \gamma \ \text{ and } \ \pi - \gamma \leqslant \theta \leqslant \pi \tag{1.2}$$